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Short communications

◎ 成田宏秋 (Hiroaki Narita)

Fourier expansion of autom. form

$G=SL_2(\mathbb{R}) \supset N=\{\begin{pmatrix}1&x\\0&1\end{pmatrix} \mid x\in\mathbb{R}\}$

$f\in L^2(\Gamma\backslash G)$: discrete subgroup $vol(\Gamma\backslash G)<\infty$

$f(ng)\in L^2(N\cap\Gamma\backslash N) \Rightarrow f(ng)=\sum_{m\in\mathbb{Z}} W_m(ng)$

$W_m(ng)=\int_{L\backslash\mathbb{R}} f(\begin{pmatrix}1&x\\0&1\end{pmatrix}g)\overline{e(mx)}\,dx$　　$L\subset\mathbb{R}$: lattice

n

Bessel fun.

What's W_m?　$\pi_f=\langle f(*g)\mid g\in SL_2(\mathbb{R})\rangle$

$\mathrm{Hom}_G(\pi_f, \mathrm{Ind}_N^G \eta_m)$　$\eta_m : N\ni\begin{pmatrix}1&x\\0&1\end{pmatrix}\mapsto e(mx)\in\mathbb{C}^\times$

$\updownarrow$

W_m

classical

What happens for other groups? (1) G : simple Lie group of tube type

i.e. G/K : tube domain & π_f : holom. D.S. with scalar min K-type.

(2) $G=Sp(1,q)$, π_f : quaternionic D.S.

(tube type case)

Thm (1) $\mathrm{Hom}_{(\mathfrak{g},K)}(\pi_f, \mathrm{Ind}_N^G\eta)$ max. nip-subgroup.

+some cond. on χ_s

$\dim=\begin{cases}1 & (\eta=\mathrm{Ind}_{N_S}^N\chi_s)\\ 0 & \text{otherwise.}\end{cases}$　　N_S : nip. radical of Siegel parabolic.

(2) ($Sp(1,q)$ case) N : 2-step (Heisenberg)　　$\kappa=\dim(\min K\text{-type})$

when $\kappa>1$

$\dim_{\mathbb{C}}\mathrm{Hom}_{(\mathfrak{g},K)}(\pi_f, \mathrm{Ind}_N^G\eta)=\begin{cases}\kappa & (\eta:\text{trivial})\\ 0 & (\eta:\text{non-trivial char.})\\ 1 & (\eta:\text{not character})\end{cases}$

∞-dim

when $\kappa=1$

$\dim_{\mathbb{C}}\mathrm{Hom}_{(\mathfrak{g},K)}(\pi_f, \mathrm{Ind}_N^G\eta)=\begin{cases}\kappa & (\eta:\text{trivial})\\ 2 & (\eta:\text{non-triv. char})\end{cases}$

Application

- Explicit construction of autom. form gen. QDS (Eisenstein-Poincaré Theta lifting)

Theta lifting $O^*(q)\to Sp(1,q)$

Kazuki Hiroe $SO(2)\to Sp(1,q)$ $q=1$

◎ 広恵一希 Whittaker vector, hypergeometric function

G : $\mathbb{R}$ reductive group. $=KAN$: 岩澤分解

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$\eta \in \hat{N}$: irr. unitary inf. dim. repr. of N $\quad \mathfrak{g} = \mathrm{Lie}\, G$

(π, H) : irr. adm. repr. of G

$\mathrm{Hom}_{(\mathfrak{g},K)}(H_{K\text{-fin}},\ C^\infty\text{-}\mathrm{Ind}_N^G \eta) \ni I$

$\mathrm{Im}(I) \ni f$ can be considered as fct. on G : generalized Whittaker fct.

(τ, V_τ) : K-type of π , $F : G \longrightarrow W_\eta \otimes V_\tau^*$: smooth.

$F(ngk) = \eta(n)\tau^*(k)F(g)$ & $D \cdot F = 0$ $\quad \forall D \in \mathcal{D}$ — system of diff. ops which characterize H_π

If π is disc. series , $\mathcal{D}$ = gradient type operators

π : principal series. $\mathcal{D} = \mathrm{Ann}_{U(\mathfrak{g})}(\pi)$ obtained by T. Oshima & generalized. for GL_n

degenerate PS induced from character. annihilator of gen. Verma mod. H. Oda & T. Oshima

$G = GL(4, \mathbb{R})$

$P_1 = P_{3,1}$, $P_2 = P_{2,2}$ $\Big)$ $\mathrm{Ind}_{P_i}^G (\mu, \nu)$

Parab.	$\hat{N}$	dim. of coadj. orbit.	
$P_{3,1}$	I	4	generic
	II	2	
$P_{2,2}$	III	2	
	IV	0	character

monomial (Bessel fu)

※

product of Bessel fun. analogus to $SL_2(\mathbb{R})$

※ Jacquet integral

$$K_2(s \mid A, B) = \int_{X \in P_2} P_s(X) \exp -\mathrm{tr}(AX + BX^{-1}) \frac{dX}{|X|^{(2+1)/2}}$$

P_2 : space of 2x2 pos. def. symm. mtr.

(A, B : 2x2 symm. matrix

$P_s(X) = X_1^{s_1} \det(X)^{s_2}$ $\quad X = \left(\begin{array}{c|c} X_1 & * \\ \hline * & * \end{array}\right)$

generalization of $K_\nu(z) = \int_0^\infty w^\nu \exp z(w + \frac{1}{w}) \frac{dw}{w}$: K-Bessel function Macdonald's function

Birne Binnegar : hypergeometric functions of matrix argument (Herz's Hypergeometric)

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ex Gauss. $\mathbb{t}=(t_1,\cdots,t_n)$

$${}_2F_1(a,b,c;\mathbb{t}) = \sum_{k=0}^{\infty}\sum_{|\lambda|=k} \frac{[a]_\lambda [b]_\lambda}{[c]_\lambda\, k!} C_\lambda(t)$$

$\lambda \in \mathrm{Par}$ partition.

Ryosuke Kodera

小寺 諒介 Repr. theory of quantized univ. env. alg, crystal basis

$U_q(\mathfrak{g})$ $\mathbb{Q}(q)$-alg. gen + rel.

$V(\lambda)$: irr. hi wt module (λ: dominant integral)

$u \in V(\lambda)_\mu$ $\quad u = \sum_{n\geq 0} f_i^{(n)} u_n \quad u_n \in \ker e_i$

$\tilde{f}_i u = \sum f_i^{(n+1)} u_n \qquad \tilde{e}_i u = \sum f_i^{(n-1)} u_n$

$A = \mathbb{Q}[q]_{(q)} = \{ \frac{g(q)}{f(q)} \in \mathbb{Q}(q) \mid f(0) \neq 0 \}$

${}^\exists L(\lambda) \subset V(\lambda)$: free A-submodule s.t. $\tilde{e}_i L(\lambda), \tilde{f}_i L(\lambda) \subset L(\lambda)$

$L(\lambda)/qL(\lambda) \supset {}^\exists B(\lambda)$: $\mathbb{Q}$-basis + some nice property; ① ②

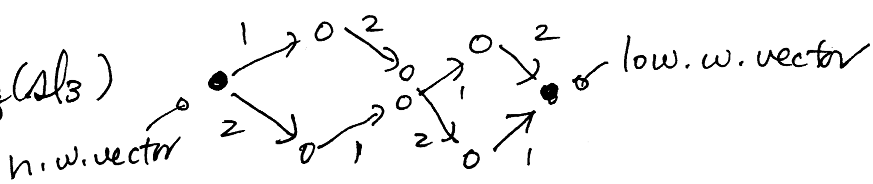
$\mathbb{Q}$-vector sp. ① $\tilde{e}_i B(\lambda) \subset B(\lambda) \cup \{0\}$, $\tilde{f}_i B(\lambda) \subset B(\lambda) \cup \{0\}$

② $b, b' \in B(\lambda)$ $\quad \tilde{e}_i b = b' \Leftrightarrow b = \tilde{f}_i b'$

$(L(\lambda), B(\lambda))$: crystal basis of $V(\lambda)$ $\rightsquigarrow$ crystal graph.

$b \xrightarrow{i} b' \overset{\text{def}}{\Longleftrightarrow} \tilde{f}_i b = b'$

ex adjoint repr. of $U_q(\mathfrak{sl}_3)$



tensor product decomp. $\quad B(\lambda) \leftrightarrow V(\lambda)$, $B(\mu) \leftrightarrow V(\mu)$) crystal basis

$\Rightarrow$ $B(\lambda) \otimes B(\mu)$: crystal basis of $V(\lambda) \otimes V(\mu)$

as a set, $B(\lambda) \times B(\mu)$

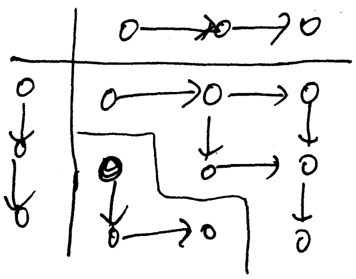
$$\tilde{f}_i(b \otimes b') = \begin{cases} \tilde{f}_i b \otimes b' & \text{if } \varphi_i(b) \geq \varepsilon_i(b') \\ b \otimes \tilde{f}_i b' & \text{if } \varphi_i(b) \leq \varepsilon_i(b') \end{cases}$$

$\varphi_i(b) = \max\{ n \mid \tilde{f}_i^{\,n} b \neq 0 \}$

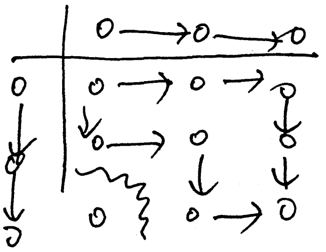
$\varepsilon_i(b) = \max\{ n \mid \tilde{e}_i^{\,n} b \neq 0 \}$

ex: $U_q(\mathfrak{sl}_3)$ $\quad V(\Lambda_1)$: 3-dim. $\quad \circ \xrightarrow{1} \circ \xrightarrow{2} \circ$

smallest possible other than the trivial



$\leftrightarrow V(\Lambda_1)\otimes V(\Lambda_1) \simeq V(2\Lambda_1)\oplus V(\Lambda_2)$



$\leftrightarrow V(\Lambda_1)\otimes V(\Lambda_2) \simeq V(\Lambda_1+\Lambda_2)\oplus V(0)$

<u>Problem</u> $\mathfrak{g}$: affine

Which fin. dim repr. has crystal base?

- 松尾清史 (Kiyoshi Matsuo)
  $\mathfrak{g}_1, \mathfrak{g}_2$: simple $\mathrm{Hom}(\mathfrak{g}_1, \mathfrak{g}_2)$
  $\mathfrak{g}_1 = sl_2$ ← Jacobson-Morozov theorem
  $\mathfrak{g}_1 = sl_3$ $\mathfrak{g}_2$ ⋯ type G_2 ⇐ same rank ⋯ reduced to the problem of Oshima
- Ugambayar.
  D-module theory and representation theory