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Hisaichi Midorikawa Date 2007 · 8 · 24

三鳥川寿一

On K-type theorem for a nonunitary principal series representation (The cases for $SU(n,1)$, $Sp(n,1)$ and $Sp(n,\mathbb{R})$)

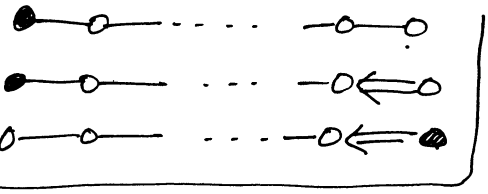
§1 $G_{\mathbb{C}} \supset G \supset K \supset B$ (compact CSG) $\quad \mathfrak{g}_{\mathbb{C}} \supset \mathfrak{g} \supset \mathfrak{k} \supset \mathfrak{b} \quad \Sigma \supset \Sigma_K$, $\ \cup\ \ \cup$, $\ P \supset P_K$

Assume P has exactly one noncompact simple root for example

$SU(n,1)$

$Sp(n,1)$

$Sp(n,\mathbb{R})$



ρ_n, ρ_K $\quad \rho = \rho_n + \rho_K$

Let μ : P-dominant integral form on $\mathfrak{b}_{\mathbb{C}}$

Put $\lambda = \mu + \rho_K - \rho_n$

Let α be a compact simple root, then α is a simple of P_K.

$1 = 2(\rho_K, \alpha)|\alpha|^{-2} = 2(\rho, \alpha)|\alpha|^{-2}$, $(\rho_n, \alpha) = 0 \Rightarrow (\lambda, \alpha) > 0$

Let δ : noncompact simple root

If $(\lambda, \delta) > 0$, then λ is P-regular $\Rightarrow$ λ is a HC-parameter of a discrete series representation.

If $(\lambda, \delta) = 0$, then λ is said to be a semiregular

$\Rightarrow$ λ is a HC-parameter of a limits of discrete series representation

In those cases, $\mu = \lambda - \rho_K - \rho_n$ is a Blattner data.

Now we assume $(\lambda, \delta) < 0$

Let $Q = MAN$ is a parabolic subgroup of G.

$M \supset M \cap B = B_-$ .... Let λ_- be the restriction of λ to $\mathfrak{b}_-$, and assume λ_- is a HC-parameter of a discrete series repr of M (or limits of disc. ser.) $\leftarrow \sigma$

My object is to study the following:

Let π_λ be a nonunitary principal series $\mathrm{ind}_Q^G(\sigma \otimes e^{\nu} \otimes \mathrm{id}) \supset \pi_\lambda$ with infinitesimal character χ_λ

Let τ_μ be the irred. unitary repr of K.

I want to calculate $(\pi_\lambda|_K : \tau_\mu)$, $\quad \pi_\lambda|_K$ = the restriction of π_λ to K.

under the following condition (*).

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Definition of the condition (*)

Let $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$: Cartan decomposition, Ad_K : the adj. repr of K on $\mathfrak{p}_{\mathbb{C}}$

Def. Let μ be the same as above $\omega \in P_n$

If $\mu+\omega$ is P_K-dominant & $(Ad_K \otimes \tau_\mu : \tau_{\mu+\omega}) > 0$,

then $2(\lambda,\omega)|\omega|^{-2} \geqq -1$ holds. $\Leftarrow$ μ is said to satisfy cond. (*)

Remark This (*) condition is one of the necessary conditions for τ_μ to be a lowest K-type of some unitary $(\mathfrak{g}, K)$-module (?)

§2 Let $G = KA_0N_0$ be Iwasawa decomp. & $\mathfrak{j}_0 = \mathfrak{a}_0 \oplus \mathfrak{b}_0$ ($\mathfrak{b}_0 \subset \mathfrak{b}$) be a maximally split Cartan subalg.

$F \subset F_0 \subset P_n$ $\quad \mathfrak{b}_- = \{H \in \mathfrak{b} \mid \alpha(H) = 0 \ \forall \alpha \in F\}$ $\quad \mathfrak{b} = \mathfrak{b}_- \oplus \mathfrak{b}_+$

max. strongly orthogonal system $\quad$ orthogonal decomp.

Then $\exists \mathfrak{j} = \mathfrak{b}_- \oplus \mathfrak{a}$: Cartan subalg. of $\mathfrak{g}$ satisfying.

$\exists x \in G_{\mathbb{C}}$ s.t. $Ad(x)\mathfrak{b}_- = \mathfrak{b}_-$ & $Ad(x)(\mathfrak{b}_+)_{\mathbb{C}} = \mathfrak{a}_{\mathbb{C}}$

Let $Q = MAN$ be a parabolic subgroup

Then $\mathfrak{m}$ has compact Cartan subalg $\mathfrak{b}_-$

Let σ be a (limits of) discrete series of M & ν : character of A

$ind_Q^G(\sigma \otimes e^{\nu+\rho_+} \otimes id)$ where $e^{2\rho_+(\log a)} = \det(Ad(a)|_N)$

§ $G = SU(n,1)$, $Sp(n,1)$, $Sp(2n,\mathbb{R})$

Case $SU(n,1)$. $P_K = \{e_i - e_j \mid 2 \leqq i < j \leqq n+1\}$, $P_n = \{e_1 - e_j \mid 2 \leqq j \leqq n+1\}$

$\delta = e_1 - e_2$ $\quad$ Let W be a Weyl group of $(\mathfrak{g}_{\mathbb{C}}, \mathfrak{b}_{\mathbb{C}})$

$w^{-1} = (1, 2, \cdots, n+1)^{-1}$ σ (?)

$Q = MAN \Leftarrow$ $F = \{e_1 - e_{n+1}\}$, $\mathfrak{b}_- = \mathfrak{b}_-(F)$, $\mathfrak{j} = \mathfrak{b}_- \oplus \mathfrak{a}$

Lemma 1 Let μ: P-dominant integral form on $\mathfrak{b}$

Then $(w\lambda - \rho)_-$ is regular or semiregular on with respect to P_-

$P_- = \{\alpha \in P \mid (\alpha, e_1 - e_{n+1}) = 0\}$

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Let $\sigma = \sigma_{(w\lambda-\rho)_-}$ be a discrete series (or limits of —) repr. of M.
Then $(\mathrm{ind}_Q^G(\sigma \otimes e^{(w\lambda-\rho)_+} \otimes id) ; \tau_\mu) = 1$ $\quad (w\lambda-\rho)_+ = (w\lambda-\rho)|_{\mathfrak{a}}$ ⌋

<u>Lemma 2</u> $G = Sp(n,1)$ we assume $\mu (\neq 0)$: P-dominant and satisfies $(*)$-condition. Let $w \in W(\mathfrak{g}_{\mathbb{C}} ; \mathfrak{b}_{\mathbb{C}})$ $\quad w = (1,2)$
$(\mathrm{ind}_Q^G(\sigma \otimes e^{(w\lambda-\rho)_+} \otimes id) : \tau_\mu) > 0$ iff $\mu = m_1 e_1 + m(e_2 + \cdots + e_n)$ and $m_1 - m = n-1$.
Moreover if $(,) > 0$ then $(,) = 1$ ⌋

<u>Case $Sp(n,\mathbb{R})$</u> $P_n = \{2e_i \mid 1 \leqq i \leqq n\} \cup \{e_i + e_j \mid 1 \leqq i < j \leqq n\}$
$\delta = 2e_n$ Assume $(\lambda, e_n) < 0$
Then $(\lambda, e_1) > (\lambda, e_2) > \cdots > (\lambda, e_n)$. Let i be the number
$\quad \overset{\vee\!\!\shortparallel}{0}$ satisfying $(\lambda, e_i) < 0$ and $(\lambda, e_{i-1}) \geqq 0$
$F = \{2e_j \mid i \leqq j \leqq n\}$ Let $\mathfrak{b}_- = \mathfrak{b}_-(F)$, $\mathfrak{a} = \mathfrak{a}(F)$ and $Q = MAN$
Then $(\lambda-\rho)_-$ is P-dominant regular (semiregular)
<u>Lemma</u> Let σ be the discrete series of M
Then $(\mathrm{ind}_Q^G(\sigma \otimes e^{(\lambda-\rho)_+} \otimes id)|_K : \tau_\mu) > 0$.
If P is maximal, then $(\sim|_K, \tau_\mu) = 1$ ⌋