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Date 2007 8 · 23

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Whittaker unitary dual of affine graded Hecke algebras.

1 Affine graded Hecke algebra

G : $\mathbb{C}$ conn reductive group $B \supset H$, W

$\mathfrak{g}, \mathfrak{b} \supset \mathfrak{z}$ $\Delta \supset \Delta^+ \supset \Pi$: roots

Defn (Lusztig '89) $\mathbb{H}$ is as a vector space $\mathbb{H} = \mathbb{C}W \otimes \mathbb{A}$, $\mathbb{A} = \mathrm{Sym}(\mathfrak{z}^*)$

subject to $s_\alpha \cdot w = s_\alpha(w)\, s_\alpha - c(\alpha)\langle w, \check{\alpha}\rangle$ $\alpha \in \Pi$, $w \in \mathfrak{z}^*$

$c : \Pi \to \mathbb{Z}_{>0}$ $c(\alpha) = c(\alpha')$ whenever $\alpha \overset{W}{\sim} \alpha'$

(Type A : Drinfeld)

$\mathbb{H}$ = assoc. graded obj. for a filtration of ideals in the affine Hecke alg.

(e.g.) Iwahori-Hecke alg s for p-adic groups

$$\overset{\text{Iwahori-Matsumoto, Borel, Casselman}}{\longleftarrow}\ \mathcal{H}\text{-module} \xrightarrow[\text{Lusztig}]{} \mathbb{H}\text{-module}$$

piece representation theory of a p-adic group

Basic case: $c : \Pi \to \mathbb{Z}_{>0}$: constant function (split p-adic; Kazhdan-Lusztig '87 Ginzburg)

c : nonconstant $\rightsquigarrow$ geometric setting

classical one, Shu Kato

non-geometrical : E. Opdam in general case.

2. $c : \Pi \to \mathbb{Z}_{>0}$ identically 1

$\mathbb{H} = \mathbb{C}W \otimes \mathbb{A}$, Bernstein, Lusztig

center $(\mathbb{H})$ = W-invariants in $\mathbb{A} = \mathrm{Sym}(\mathfrak{z}^*)$

characters $\longleftrightarrow$ W-orbits in $\mathfrak{z}$, χ

$\mathbb{H}\text{-mod}_\chi$ = (central char $= \chi$ modules)

Geometry $\mathfrak{g}_{1,\chi}$ = (+1) eigenspace of $\mathrm{ad}(\chi)$ on $\mathfrak{g}$

$G_{0,\chi}$ = centralizer of χ in G

$G_{0,\chi} \backslash \mathfrak{g}_{1,\chi}$: Kawanaka ... finitely many, $\exists$ unique open orbit

$\check{\rho} = \frac{1}{2}\sum_{\alpha \in \Delta^+} \alpha^\vee$

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Theorem (Lusztig.) In $\mathbb{H}$-mod_χ

the standard modules $\longleftrightarrow \left\{ (\mathcal{O}, \psi) : \mathcal{O} \in G_{0,x} \backslash \mathfrak{g}_{1,x} \right.$
(the irred. modules) $\qquad \psi$: $G_{0,x}$-equt localsystem on $\mathcal{O}$ $\left.\right\}$

$L(\chi, \mathcal{O}, \psi) \qquad X(\chi, \mathcal{O}, \psi) \qquad \psi \in \widehat{A_G(e, \chi)}$: comp. group. $e \in \mathcal{O}$

of "Springer type"

Theorem (Ginzburg ... affine $\mathcal{H}$; Lusztig ... graded $\mathbb{H}$)

[not all the pair appear. only some subset will appear and they are called Springer type (O, trivial) is Springer type.]

In the Grothendieck group of $\mathbb{H}$-mod_χ

$$X(\mathcal{O}, \psi) = \sum_{(\mathcal{O}', \psi')} P_{(\mathcal{O}, \psi), (\mathcal{O}', \psi')}(1) \, L(\mathcal{O}', \psi')$$

where $P_{(\mathcal{O}, \psi), (\mathcal{O}', \psi')}(q) = \sum [\psi : \mathbb{H}^{2i}(\overline{\mathcal{O}'}, \psi')|_{\mathcal{O}}] q^i$

is the K-L polynomial.

Corollaries (1) $P_{(\mathcal{O}, \psi), (\mathcal{O}', \psi')} = 0$ unless $\mathcal{O} \subset \overline{\mathcal{O}'}$

(2) If $\mathcal{O}$ is the open orbit in $\mathfrak{g}_{1,x}$, then $X(\mathcal{O}, \psi) = L(\mathcal{O}, \psi)$

(3) $\chi = \check{\rho}$, all nontrivial polys are 1.

Lusztig '06 algorithm to compute these polynomials.

3. Reducibility of standard modules

Langlands data is generic ($\mathbb{C}W \subset \mathbb{H}$)

Defn V generic if $V|_W$ contains the sgn reprn.

Equivalently $V = X(\chi, \mathcal{O}_{op}^{\chi}, \text{triv.})$
(Barbasch-Moy)

Marc Reeder: these are "Iwahori-fixed vectors" of generic representations of split p-adic groups

When is a parabolically induced module: $X(P, \sigma, \nu) = \mathbb{H} \otimes_{\mathbb{H}_P} (\sigma \otimes \mathbb{C}_\nu)$

σ: tempered. $\leftrightarrow (\chi, \mathcal{O}_{op}, \psi)$ where $\chi = \frac{h}{2}$ + ellipticss $\qquad$ Characte

$\mathcal{O}_{op} \subset$ nilpotent cone $\qquad \Pi_P \subset \Pi$

$\overset{\psi}{e} \rightsquigarrow \{e, h, f\}$: $\mathfrak{sl}_2$-triple

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$\left.\begin{array}{l}\nu : \text{real, dominant} \\ \chi = h/2\end{array}\right) \longrightarrow \exists 1$ irred. quotient in $X(P, \sigma, \nu)$

Standard technique (Speh-Vogan '86) reduce to P: max. parabolic

$P = m + n$: parabolic $\qquad \sigma \leftrightarrow sl(2) \hookrightarrow m$

$n \cdots sl_2$-module $\qquad n = \bigoplus_{i=1}^{k} n_i$ coweight decomp. wrt $\omega = \alpha^\vee$

$\{\alpha\} = \Pi - \Pi_P$

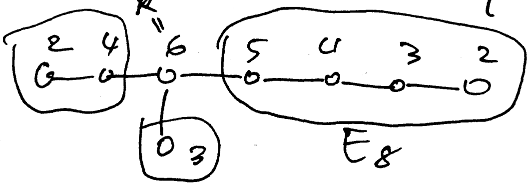
$n_i = \bigoplus_i (d_{ij})$: irred. decomp. by sl_2

dimension

Muić-Shahidi, Barbasch-Ciubotaru

<u>Theorem</u> (M-S, B-C)

σ: generic tempered module. Then the reducibility points $\nu > 0$ of $X(P, \sigma, \nu)$ are $\left\{\frac{d_{ij}+1}{2i}\right\}_{i,j}$ $\lrcorner$



$A_4 + A_2 + A_1 \subset E_8$

reducibility pts. $\left\{\frac{3}{10}, \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, 1, \frac{7}{6}, \cdots\right\}$

only case when the first red. pt is not $\frac{1}{i_0(P)}$

discrete series

unitary modules: $X(P, \sigma, \nu)$ $\nu = 0$ irreducible

$L(P, \sigma, \nu)$

$0 \qquad \frac{3}{10} \qquad 1/2$

4 Answer unitary generic dual

$\mathcal{O}$: nilpotent G-orbit in $\mathfrak{g}$

$CS(\mathcal{O}) = \{x : \text{real } X(x, \mathcal{O}^x_{op}, \text{triv.}) \text{ is unitary and } \mathcal{O}^x_{op} \subset \mathcal{O}\}$

complementary series

Describe $CS(\mathcal{O})$ for every $\mathcal{O}$.

1. extreme case is $\mathcal{O}=0$

$CS(0)$ = unitary generic and sph. modules

D. Vogan G_2 real (1992)

$X(x, 0, \mathrm{triv})$ irred.

(full) spherical principal series