Harmonic polynomial of Cartan type in 3 variables.

dimension of harmonic polynomial of degree $1 = 3$:

x_1
x_2
x_3

dimension of harmonic polynomial of degree $2 = 3$:

$x_1 x_2$
$x_1 x_3$
$x_2 x_3$

dimension of harmonic polynomial of degree $3 = 4$:

$(x_1 - x_2)\, x_1 x_2$
$(x_1 - x_3)\, x_1 x_3$
$(x_2 - x_3)\, x_2 x_3$
$x_1 x_2 x_3$

dimension of harmonic polynomial of degree $4 = 2$:

$(x_1 - x_2)\, x_1 x_2 x_3$
$(x_1 - x_3)\, x_1 x_2 x_3$

dimension of harmonic polynomial of degree $5 = 2$:

$(x_1 - x_2)\, (x_1 + x_2 - 3\, x_3)\, x_1 x_2 x_3$
$(x_1 - x_3)\, (x_1 - 3\, x_2 + x_3)\, x_1 x_2 x_3$

dimension of harmonic polynomial of degree $6 = 1$:

$(x_2 - x_3)\, (x_1 - x_3)\, (x_1 - x_2)\, x_1 x_2 x_3$

Harmonic polynomial of Cartan type in 4 variables.

dimension of harmonic polynomial of degree $1 = 4$:

x_1
x_2
x_3
x_4

dimension of harmonic polynomial of degree $2 = 6$:

x_3x_4
x_1x_3
x_1x_2
x_2x_3
x_1x_4
x_2x_4

dimension of harmonic polynomial of degree $3 = 10$:

$(x_1 - x_2)\, x_1x_2$
$(x_1 - x_3)\, x_1x_3$
$(x_2 - x_3)\, x_2x_3$
$(x_1 - x_4)\, x_1x_4$
$(x_2 - x_4)\, x_2x_4$
$(x_3 - x_4)\, x_3x_4$
$x_1x_2x_3$
$x_1x_2x_4$
$x_1x_3x_4$
$x_2x_3x_4$

dimension of harmonic polynomial of degree $4 = 9$:

$(x_2 - x_4)\, x_2x_3x_4$
$(x_1 - x_2)\, x_1x_2x_3$
$(x_1 - x_2)\, x_1x_2x_4$
$(x_1 - x_3)\, x_1x_3x_4$
$(x_2 - x_3)\, x_2x_3x_4$
$(x_1 - x_4)\, x_1x_2x_4$
$(x_1 - x_3)\, x_1x_2x_3$
$(x_1 - x_4)\, x_1x_3x_4$
$x_1x_2x_3x_4$

dimension of harmonic polynomial of degree $5 = 11$:

$(x_1 - x_2)\, x_1x_2x_3x_4$
$(x_1 - x_3)\, x_1x_2x_3x_4$
$(x_1 - x_4)\, x_1x_2x_3x_4$
$(x_1 - x_3)\, (x_1 + x_3 - 3\, x_4)\, x_1x_3x_4$
$(x_2 - x_3)\, (x_2 + x_3 - 3\, x_4)\, x_2x_3x_4$
$(x_1 - x_4)\, (x_1 - 3\, x_3 + x_4)\, x_1x_3x_4$
$(x_2 - x_4)\, (x_2 - 3\, x_3 + x_4)\, x_2x_3x_4$
$(x_1 - x_4)\, (x_1 - 3\, x_2 + x_4)\, x_1x_2x_4$
$(x_1 - x_2)\, (x_1 + x_2 - 3\, x_3)\, x_1x_2x_3$
$(x_1 - x_3)\, (x_1 - 3\, x_2 + x_3)\, x_1x_2x_3$
$(x_1 - x_2)\, (x_1 + x_2 - 3\, x_4)\, x_1x_2x_4$

dimension of harmonic polynomial of degree $6 = 9$:

$(x_2 - x_3)\, (x_1 - x_4)\, x_1x_2x_3x_4$
$(x_2 - x_4)\, (x_1 - x_3)\, x_1x_2x_3x_4$
$(x_1 - x_4)\, (x_1 - 3\, x_2 + x_4)\, x_1x_2x_3x_4$
$(x_2 - x_4)\, (x_2 - 3\, x_3 + x_4)\, x_1x_2x_3x_4$
$(x_3 - x_4)\, (3\, x_2 - x_3 - x_4)\, x_1x_2x_3x_4$
$(x_2 - x_3)\, (x_1 - x_3)\, (x_1 - x_2)\, x_1x_2x_3$
$(x_3 - x_4)\, (x_2 - x_4)\, (x_2 - x_3)\, x_2x_3x_4$
$(x_2 - x_4)\, (x_1 - x_4)\, (x_1 - x_2)\, x_1x_2x_4$
$(x_3 - x_4)\, (x_1 - x_4)\, (x_1 - x_3)\, x_1x_3x_4$

dimension of harmonic polynomial of degree $7 = 6$:

$(x_2 - x_4)\, (3\, x_1 - x_2 - x_4)\, (x_1 + x_2 - 5\, x_3 + x_4)\, x_1x_2x_3x_4$
$(x_1 - x_4)\, (x_1 - 3\, x_2 + x_4)\, (x_1 + x_2 - 5\, x_3 + x_4)\, x_1x_2x_3x_4$
$(x_3 - x_4)\, (3\, x_1 - x_3 - x_4)\, (x_1 - 5\, x_2 + x_3 + x_4)\, x_1x_2x_3x_4$
$(x_2 - x_4)\, (x_1 - x_4)\, (x_1 - x_2)\, x_1x_2x_3x_4$
$(x_3 - x_4)\, (x_1 - x_4)\, (x_1 - x_3)\, x_1x_2x_3x_4$
$(x_3 - x_4)\, (x_2 - x_4)\, (x_2 - x_3)\, x_1x_2x_3x_4$

dimension of harmonic polynomial of degree $8 = 5$:

$(x_2 - x_3)\, (x_1 - x_3)\, (x_1 - x_2)\, (x_1 + x_2 + x_3 - 6\, x_4)\, x_1x_2x_3x_4$
$(x_3 - x_4)\, (x_2 - x_4)\, (x_2 - x_3)\, (6\, x_1 - x_2 - x_3 - x_4)\, x_1x_2x_3x_4$
$(x_3 - x_4)\, (x_1 - x_4)\, (x_1 - x_3)\, (x_1 - 6\, x_2 + x_3 + x_4)\, x_1x_2x_3x_4$
$(x_2 - x_4)\, (x_1 - x_3)\, \left(5\, x_1x_2 - 15\, x_1x_3 + 5\, x_1x_4 - 3\, x_2^2 + 5\, x_2x_3 + 5\, x_3x_4 - 3\, x_4^2\right) x_1x_2x_3x_4$
$(x_2 - x_3)\, (x_1 - x_4)\, \left(5\, x_1x_2 + 5\, x_1x_3 - 15\, x_1x_4 - 3\, x_2^2 + 5\, x_2x_4 - 3\, x_3^2 + 5\, x_3x_4\right) x_1x_2x_3x_4$

dimension of harmonic polynomial of degree $9 = 3$:

$$(x_2 - x_3)\,(x_1 - x_3)\,(x_1 - x_2)\,\Big(x_1^2 - 14\,x_1x_2 - 14\,x_1x_3 + 35\,x_1x_4 + x_2^2 - 14\,x_2x_3 \\ +35\,x_2x_4 + x_3^2 + 35\,x_3x_4 - 70\,x_4^2\Big)\,x_1x_2x_3x_4$$

$$(x_3 - x_4)\,(x_1 - x_4)\,(x_1 - x_3)\,\Big(x_1^2 + 35\,x_1x_2 - 14\,x_1x_3 - 14\,x_1x_4 - 70\,x_2^2 + 35\,x_2x_3 \\ +35\,x_2x_4 + x_3^2 - 14\,x_3x_4 + x_4^2\Big)\,x_1x_2x_3x_4$$

$$(x_3 - x_4)\,(x_2 - x_4)\,(x_2 - x_3)\,\Big(70\,x_1^2 - 35\,x_1x_2 - 35\,x_1x_3 - 35\,x_1x_4 - x_2^2 \\ +14\,x_2x_3 + 14\,x_2x_4 - x_3^2 + 14\,x_3x_4 - x_4^2\Big)\,x_1x_2x_3x_4$$

dimension of harmonic polynomial of degree $10 = 1$:

$$(x_3 - x_4)\,(x_2 - x_4)\,(x_2 - x_3)\,(x_1 - x_4)\,(x_1 - x_3)\,(x_1 - x_2)\,x_1x_2x_3x_4$$